MA26500: SAMPLE EXAM II **NAME:** ______________________

1. The subspace of $\mathbb{R}^3$ spanned by $\{(1,2,3),(1,3,5),(1,5,k)\}$ has dimension 3 if

A. $k \neq 1$

B. $k \neq 9$

C. $k = 0$

D. $k = 1$

E. $k = 9$

2. Consider the the vectors $v_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ 1 \\ 7 \\ 3 \end{bmatrix}$, $v_3 = \begin{bmatrix} 5 \\ -3 \\ 9 \\ 1 \end{bmatrix}$, and $v_4 = \begin{bmatrix} -2 \\ 4 \\ 2 \\ 8 \end{bmatrix}$. The dimension of the space $\text{span}\{v_1, v_2, v_3, v_4\}$ is then equal to

A. 1

B. 2

C. 3

D. 4

E. 5

3. The subset $S = \{A \in M_3(\mathbb{R}^3) : A^T = -A\}$ of the space of 3×3 matrices with real elements is a vector space such that

A. dim$S$=2 and a basis for S is $\{ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \}$

B. dim$S$=2 and a basis for S is $\{ \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \}$

C. dim$S$=3 and a basis for S is $\{ \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix} \}$

D. dim$S$=3 and a basis for S is $\{ \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 2 & 0 \end{bmatrix} \}$

E. None of the above.

4. Suppose that the $n \times n$ matrix A is invertible. Which of the following statements are true?

(i) The reduced row echelon form of A is I_n

(ii) $\det A = 0$

(iii) The row space of A is $\mathbb{R}^n$.

(iv) The column vectors of A are linearly dependent.

(v) For every vector $b \in \mathbb{R}^n$, the system $Ax = b$ has a unique solution.

A. (i), (ii), (iii)

B. (iii), (iv), (v)

C. (ii), (iv)

D. (i), (iii), (iv)

E. (i), (iii), (v)

5. Consider the following inconsistent linear system $Ax = b$, where

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 3 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

The *least squares* solution of the linear system is

A. $\hat{x} = \begin{bmatrix} 1 \\ 1/7 \end{bmatrix}$

B. $\hat{x} = \begin{bmatrix} 1/7 \\ 0 \end{bmatrix}$

C. $\hat{x} = \begin{bmatrix} 1/7 \\ 1/7 \end{bmatrix}$

D. $\hat{x} = \begin{bmatrix} -1 \\ 1/7 \end{bmatrix}$

E. $\hat{x} = \begin{bmatrix} 0 \\ -1/7 \end{bmatrix}$

6. Let $x, y \in \mathbb{R}^2$ be two vectors, satisfying the following properties:

(i) $x \cdot y = 0$.

(ii) $\|x\| = 2$ and $\|y\| = 1$.

Then, for real numbers a, b, what is the expression for $\|ax + by\|^2$?

A. $a^2 + b^2$

B. $2a^2 + b^2$

C. $4a^2 + b^2$

D. $4a^2 + 4ab + b^2$

E. $a^2 + 4ab + b^2$

7. Let

$$v_1 = \begin{bmatrix} 1 & 3 & 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 & -2 & 1 \end{bmatrix}$$

be vectors in $\mathbb{R}^3$ with the standard inner product. Let W be the subspace of $\mathbb{R}^3$ spanned by $\{v_1, v_2\}$. A basis for $W^\perp$ is

A. $\begin{bmatrix} 0 & -1 & 3 \end{bmatrix}$

B. $\begin{bmatrix} 5 & -2 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 3 & 1 \end{bmatrix}, \begin{bmatrix} -1 & -2 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 4 & -1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$

E. $\begin{bmatrix} -2 & 1 & -1 \end{bmatrix}, \begin{bmatrix} 3 & -1 & 1 \end{bmatrix}$

8. Given the set of linearly independent vectors in $\mathbb{R}^3$ with the standard inner product

$$u_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}.$$

Use the Gram Schmidt process to turn it into an orthonromal basis $\{w_1, w_2\}$. We have

A. $w_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 4 \\ 4 \\ -3 \end{bmatrix}$

B. $w_1 = \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

C. $w_1 = \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 2/\sqrt{5} \\ 0 \\ 1/\sqrt{5} \end{bmatrix}$

D. $w_1 = \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 3/\sqrt{14} \\ 2/\sqrt{14} \\ -1/\sqrt{14} \end{bmatrix}$

E. $w_1 = \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 2/\sqrt{13} \\ 0 \\ -3/\sqrt{13} \end{bmatrix}$

9. Determine all values of k such that the vectors $(1, -1, 0)$, $(1, 2, 2)$, $(0, 3, k)$ are a basis for $\mathbb{R}^3$.

A. $k = 1$

B. $k = 2$

C. $k \neq 2$

D. $k \neq 1$

E. $k \neq 3$

10. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by $T(\begin{bmatrix}1\\1\end{bmatrix}) = \begin{bmatrix}1\\2\\3\end{bmatrix}$ and $T(\begin{bmatrix}1\\0\end{bmatrix}) = \begin{bmatrix}0\\3\\1\end{bmatrix}$.

What is $T(\begin{bmatrix}1\\2\end{bmatrix})$?

A. $\begin{bmatrix}4\\6\\3\end{bmatrix}$

B. $\begin{bmatrix}1\\0\\1\end{bmatrix}$

C. $\begin{bmatrix}2\\2\\8\end{bmatrix}$

D. $\begin{bmatrix}1\\5\\4\end{bmatrix}$

E. $\begin{bmatrix}2\\1\\5\end{bmatrix}$